Linguaggi di Programmazione: Semantica
2011-02-02

<u>Nota</u>: svolgere questo esercizio su un foglio separato.

Esercizio 3. Si considerino i tipi

$$\tau_1 = \mathsf{nat} \to (\mathsf{nat} \to \mathsf{nat})$$
$$\tau_2 = (\mathsf{nat} \times \mathsf{nat}) \to \mathsf{nat}$$

e le espressioni del λ-calcolo tipato lazy

$$e_{12} = \lambda fc.\, f(\pi_1(c))(\pi_2(c))$$
$$e_{21} = \lambda gxy.\, g(x,y)$$

Si noti che $e_{12} : \tau_1 \to \tau_2$, e che $e_{21} : \tau_2 \to \tau_1$.

Siano $v_1 \in (V_{\tau_1})_\perp$ e $v_2 \in (V_{\tau_1})_\perp$ tali che $v_1 \neq \perp, v_2 \neq \perp$. Determinare **se** le seguenti equazioni valgono per ogni v_1, v_2, ρ, o se invece non valgono per qualche v_1, v_2, ρ :

$$[\![\, e_{21}(e_{12}\ f)\,]\!]^{l-\mathsf{CON}} (\rho[f = v_1]) = v_1$$
$$[\![\, e_{12}(e_{21}\ g)\,]\!]^{l-\mathsf{CON}} (\rho[g = v_2]) = v_2$$

dove $\rho[x = v]$ indica l'aggiornamento di ambiente.

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Consider the following types

$$\tau_1 = \mathsf{nat} \to (\mathsf{nat} \to \mathsf{nat})$$
$$\tau_2 = (\mathsf{nat} \times \mathsf{nat}) \to \mathsf{nat}$$

and the following expression in the lazy typed λ-calculus

$$e_{12} = \lambda fc.\, f(\pi_1(c))(\pi_2(c))$$
$$e_{21} = \lambda gxy.\, g(x,y)$$

Note that $e_{12} : \tau_1 \to \tau_2$, and $e_{21} : \tau_2 \to \tau_1$.

Let $v_1 \in (V_{\tau_1})_\perp$ and $v_2 \in (V_{\tau_1})_\perp$ be such that $v_1 \neq \perp, v_2 \neq \perp$.

State **whether** the following equations hold for all v_1, v_2, ρ, or, instead, they do not hold for some v_1, v_2, ρ :

$$[\![\, e_{21}(e_{12}\ f)\,]\!]^{l-\mathsf{CON}} (\rho[f = v_1]) = v_1$$
$$[\![\, e_{12}(e_{21}\ g)\,]\!]^{l-\mathsf{CON}} (\rho[g = v_2]) = v_2$$

where $\rho[x = v]$ denotes the environment update.