



Mathematical Logic - 2015

Propositional Logic: exercises

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Truth Tables Example (2)

Use the truth tables method to determine whether $(\neg p \vee q) \wedge (q \rightarrow \neg r \wedge \neg p) \wedge (p \vee r)$ (denoted with F) is satisfiable.

p	q	r	$\neg p \vee q$	$\neg r \wedge \neg p$	$q \rightarrow \neg r \wedge \neg p$	$(p \vee r)$	F
T	T	T	T	F	F	T	F
T	T	F	T	F	F	T	F
T	F	T	F	F	T	T	F
T	F	F	F	F	T	T	F
F	T	T	T	F	F	T	F
F	T	F	T	T	T	F	F
F	F	T	T	F	T	T	T
F	F	F	T	T	T	F	F

There exists an interpretation satisfying F , thus F is satisfiable.

Truth Tables Example (3)

Use the truth tables method to determine whether $p \wedge \neg q \rightarrow p \wedge q$ is a logical consequence of $\neg p$.

p	q	$\neg p$	$p \wedge \neg q$	$p \wedge q$	$p \wedge \neg q \rightarrow p \wedge q$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	F	T
F	F	T	F	F	T

Truth Tables Example (4)

Use the truth tables method to determine whether $p \rightarrow (q \wedge \neg q)$ and $\neg p$ are logically equivalent.

p	q	$q \wedge \neg q$	$p \rightarrow (q \wedge \neg q)$	$\neg p$
T	T	F	F	F
T	F	F	F	F
F	T	F	T	T
F	F	F	T	T

Truth Tables Exercises

Compute the truth tables for the following propositional formulas:

- $(p \rightarrow p) \rightarrow p$
- $p \rightarrow (p \rightarrow p)$
- $p \vee q \rightarrow p \wedge q$
- $p \vee (q \wedge r) \rightarrow (p \wedge r) \vee q$
- $p \rightarrow (q \rightarrow p)$
- $(p \wedge \neg q) \vee \neg(p \leftrightarrow q)$

Truth Tables Exercises

Use the truth table method to verify whether the following formulas are valid, satisfiable or unsatisfiable:

- $(p \rightarrow q) \wedge \neg q \rightarrow \neg p$
- $(p \rightarrow q) \rightarrow (p \rightarrow \neg q)$
- $(p \vee q \rightarrow r) \vee p \vee q$
- $(p \vee q) \wedge (p \rightarrow r \wedge q) \wedge (q \rightarrow \neg r \wedge p)$
- $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$
- $(p \vee q) \wedge (\neg q \wedge \neg p)$
- $(\neg p \rightarrow q) \vee ((p \wedge \neg r) \leftrightarrow q)$
- $(p \rightarrow q) \wedge (p \rightarrow \neg q)$
- $(p \rightarrow (q \vee r)) \vee (r \rightarrow \neg p)$

Problem formalization (1)

Formalize the following propositions and verify whether the third proposition is a logical consequence of the first and the second: *"If you play and you study you'll pass the exams, while if you play and don't study you won't pass. Thus, if you play, either you study and you'll pass the exams, or you don't study and you won't pass."*

$$L = \{P, S, E\}$$

$P =$ "you play" , $S =$ "you study" ; $E =$ "you pass the exam"

1. $(P \wedge S) \rightarrow E$
2. $(P \wedge \neg S) \rightarrow \neg E$
3. $P \rightarrow (S \wedge E) \vee (\neg S \wedge \neg E)$

We need to prove that $1 \wedge 2 \models 3$

Use truth tables

Problem formalization (2)

Brown, Jones, and Smith are suspected of a crime.
They testify as follows:

Brown: *“Jones is guilty and Smith is innocent”*.

Jones: *“If Brown is guilty then so is Smith”*.

Smith: *“I'm innocent, but at least one of the others is guilty”*.

Let B, J, and S be the statements “Brown is guilty”, “Jones is guilty”, and “Smith is guilty”, respectively. Do the following:

1. Express the testimony of each suspect as a PL formula.

The three statements can be expressed as $J \wedge \neg S$, $B \rightarrow S$, and $\neg S \wedge (B \vee J)$

Problem formalization (2)

1. Write a truth table for the three testimonies.

	B	J	S	$J \wedge \neg S$	$B \supset S$	$\neg S \wedge (B \vee J)$
(1)	T	T	T	F	T	F
(2)	T	T	F	T	F	T
(3)	T	F	T	F	T	F
(4)	T	F	F	F	F	T
(5)	F	T	T	F	T	F
(6)	F	T	F	T	T	T
(7)	F	F	T	F	T	F
(8)	F	F	F	F	T	F

Problem formalization (2)

Use the truth table to answer the following questions:

(a) Are the three testimonies satisfiable?

Yes, assignment (6) makes them all true

(b) The testimony of one of the suspects follows from that of another. Which from which?

$$J \wedge \neg S \models \neg S \wedge (B \vee J)$$

(c) Assuming that everybody is innocent, who committed perjury?

Everybody is innocent corresponds to assignment (8), and in this case the statements of Brown and Smith are false.

(d) Assuming that all testimonies are true, who is innocent and who is guilty?

Assuming that all testimonies are true corresponds to assignment (6). In this case Jones is guilty and the others are innocents.

Problem formalization (3)

Problem

Kyle, Neal, and Grant find themselves trapped in a dark and cold dungeon (HOW they arrived there is another story). After a quick search the boys find three doors, the first one red, the second one blue, and the third one green.

Behind one of the doors is a path to freedom. Behind the other two doors, however, is an evil fire-breathing dragon. Opening a door to the dragon means almost certain death.

On each door there is an inscription:

freedom	freedom	freedom
is behind	is not behind	is not behind
this door	this door	the blue door

Given the fact that at **LEAST ONE** of the three statements on the three doors is true and at **LEAST ONE** of them is false, which door would lead the boys to safety?

Problem formalization (3)

r : "freedom is behind the red door"

b : "freedom is behind the blue door"

g : "freedom is behind the green door"

"behind one of the door is a path to freedom, behind the other two doors is an evil dragon"

$$(r \wedge \neg b \wedge \neg g) \vee (\neg r \wedge b \wedge \neg g) \vee (\neg r \wedge \neg b \wedge g)$$

"at least one of the three statements is true"

$$r \vee \neg b$$

"at least one of the three statements is false"

$$\neg r \vee b$$

Freedom is behind the
green door!

r	b	g	2	3	$2 \wedge 3$
T	F	F	T	F	F
F	T	F	F	T	F
F	F	T	T	T	T