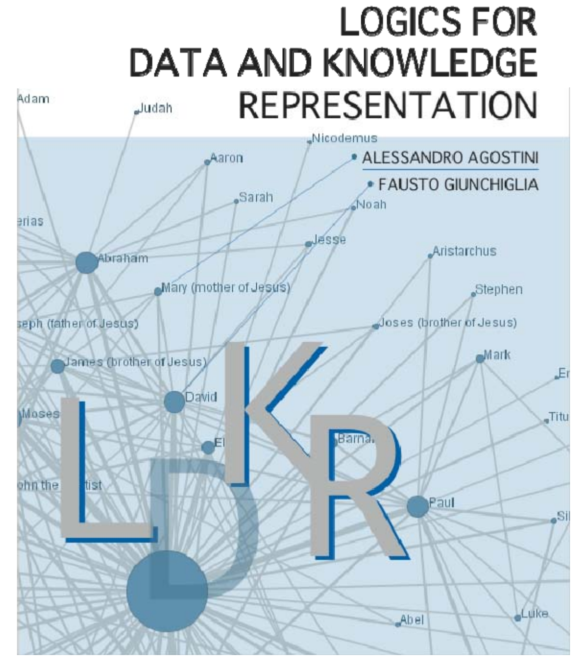

Logics for Data and Knowledge Representation

Exercises: DL

DL family of languages

AL

<Atomic> ::= A | B | ... | P | Q | ... | ⊥ | ⊤

<wff> ::= <Atomic> | ¬<Atomic> | <wff> ⊓ <wff> | ∀R.C | ∃R.⊤

ALU <wff> ⊔ <wff>

ALE ∃R.C

ALN ≥nR | ≤nR

ALC ¬ <wff>

FL- *is AL* with the elimination of ⊤, ⊥ and ¬

FL0 *is FL-* with the elimination of ∃R.⊤

DL family of languages

- Examples of formulas in each of the languages:

Formula	$\mathcal{AL}$	$\mathcal{ALU}$	$\mathcal{ALE}$	$\mathcal{ALN}$	$\mathcal{ALC}$
¬A	☑	☑	☑	☑	☑
A⊔B		☑			
∃R.C			☑		
≥2R				☑	
¬(A⊓B)		☑ ¬A⊔¬B	☑ ¬A⊔¬B		☑

DL semantics

- Verify the following equivalences hold for all interpretations (Δ, I):
  - $I(\neg(C \sqcap D)) = I(\neg C \sqcup \neg D)$
  - $I(\neg(C \sqcup D)) = I(\neg C \sqcap \neg D)$
  - $I(\neg\forall R.C) = I(\exists R.\neg C)$
  - $I(\neg\exists R.C) = I(\forall R.\neg C)$

- $I(\neg\exists R.C) = \{a \in \Delta \mid \text{not exists } b \text{ s.t. } (a,b) \in I(R), b \in I(C)\}$

  $= \{a \in \Delta \mid \neg \exists b.\ R(a,b) \wedge C(b)\}$

  $= \{a \in \Delta \mid \forall b.\ \neg (R(a,b) \wedge C(b))\}$

  $= \{a \in \Delta \mid \forall b.\ R(a,b) \rightarrow \neg C(b)\}$

  $= I(\forall R.\neg C)$

Reasoning services: Consistency

- An ABox A is consistent with respect to a TBox T if there is an interpretation I which is a model of both A and T.

  T = {Parent $\sqsubseteq \leq 1$ hasChild.Person}

  A = {hasChild(mary, bob), hasChild(mary, cate)}

  A is consistent ALONE but not consistent with respect T.

  In fact, from A mary has two children while T imposes maximum one

Reasoning

Given the TBox and ABox below

- T:
  - $Female \sqsubseteq Human$
  - $Male \sqsubseteq Human$
  - $Mother \sqsubseteq Female$
  - $Father \sqsubseteq Male$
  - $Child \equiv \exists has.Mother \sqcap \exists has.Father$
  - $Male \sqcap Female \sqsubseteq \bot$
- A:
  - Mother(Anna)
  - Father(Bob)
  - has(Cate,Anna)
  - has(Cate,Bob)

Prove:

1. Human(Anna)
2. $\neg$Female(Bob)
3. Child(Cate)

Reasoning

Expand A w.r.t. T:

- A:
  - Mother(Anna) ⇨ Female(Anna) ⇨ Human(Anna)
  - Father(Bob) ⇨ Male(Bob) ⇨ Human(Bob) , ¬Female(Bob)
  - has(Cate,Anna) ⇨ Child(Cate)
  - has(Cate,Bob) ⇨ Child(Cate)

Reasoning using Tableau calculus

- Given the ABox A = {hasParent(Speedy, Furia)},

  prove with Tableau algorithm the satisfiability of the following formula:

  ∃Parent.Horse ⊓ ¬ (Horse ⊓ Mule)

  Both ∃Parent.Horse and ¬ (Horse ⊓ Mule) have to be satisfied ⊓-rule

  (1) ∃hasParent.Horse ⇨ A'' = A' ∪ {Horse(Furia)} ∃-rule

  (2) ¬ (Horse ⊓ Mule) ⇨ ¬ Horse ⊔ ¬ Mule ⇨
  It is not consistent for A' = A ∪ {¬ Horse(Furia)} ⊔-rule
  (backtracking)
  It is consistent for A' = A ∪ {¬ Mule(Furia)} ⊔-rule