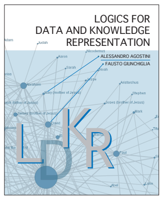
Logics for Data and Knowledge Representation

Alessandro Agostini
agostini@dit.unitn.it

Fausto Giunchiglia
fausto@dit.unitn.it

University of Trento

The Logic of Contexts



- Introduction
  - Contexts
- Syntax
- Semantics
  - Local Models

Contexts

- The notion of "context" is used in various areas of AI, including data and knowledge representation, NLP, and multimedia IR.
- Although this,
  - its meaning is frequently left to the user;
  - its use is implicit and intuitive;
  - its formalization is poor or missing.

Example (very famous)

- (Tarski, 1931) Consider the proposition

  'snow is white'

  - Is this proposition true?
  - What about the colour of the snow on top of Mount Etna in Sicily?

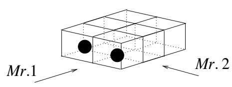
(Mount Etna is one of the most active volcanoes in the world...)

What is a Context?

- A context "surrounds, and gives meaning to, something else" (*Webster*)
  - In linguistic: it is "the text surrounding a term in which the term is used."
  - In logic (Giunchiglia, 1993): that subset of the complete state of an individual that is used for reasoning about a given goal
    - e.g. formulate a query.

Subjective Perspective ("magic box", 1998)

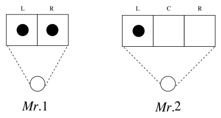
- Intuitively, a context is a theory of the world which encodes (formally by using a logic called contextual logic--CxL) an individual's *subjective perspective* about the world.



(Giunchiglia & Ghidini, KR-98; "magic box" due to L. Serafini)

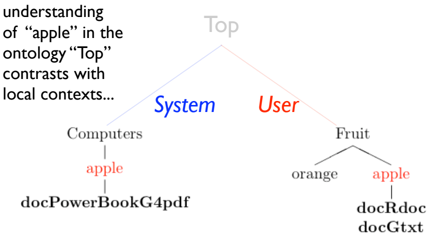
Subjective Perspective (“magic box”, 1998)

- The “magic box” represents a world with two contexts, i.e., local models that encode Mr.1 and Mr. 2’s subjective view of a domain.



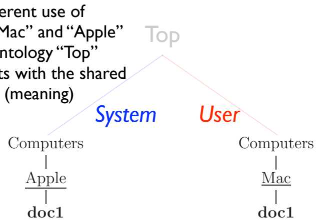
Example

- The shared understanding of “apple” in the ontology “Top” contrasts with local contexts...



Example

- The different use of terms “Mac” and “Apple” in the ontology “Top” contrasts with the shared context (meaning)



Contextual Reasoning

- F. Giunchiglia was the first to shift the focus explicitly from contexts to contextual reasoning in his 1993 seminal paper titled “Contextual Reasoning”.
- His main motivation was the problem of locality, namely the problem of modelling reasoning which uses only a subset of what reasoners actually know about the world.

Basic Intuition

- We never consider all we know but rather a very small subset of it.
  - This is the **Principle of Locality**.
- Locality of reasoning, that is the small subset of knowledge be considered is what determines the context of reasoning.

Language

- The first step in setting up a formal language (viz. a contextual language L) is to list the symbols, that is, the alphabet of symbols.
- For every $i \in N$, we define an alphabet Σ_i for a contextual language L_i such that $L= \{L_i\}_{i \in N}$
- Similarly to any logical language, we can divide symbols in Σ_i in ‘descriptive’ (nonlogical) and ‘non-descriptive’ (logical).

Multi-Context Alphabet

- We fix a denumerable set I of indices, each index represents the names of a context.
- Definition. A multicontext alphabet is a set

  $\Sigma = \bigcup_{i \in I} \Sigma_i$ (note: $I \subseteq N$ natural numbers)

  where Σ_i is a first-order alphabet enriched by some auxiliary symbols to build contextual formulas (see the next slides).

From Σ_i to the Multi-Context Language

- From $\Sigma = \bigcup_{i \in I} \Sigma_i$ we define a family $\{L_i\}_{i \in I}$.
- Definition. A family of sets over I is a set with repetitions, i.e., a set where each element S_i can occur infinitely many often.
  - In symbols: $\{S_i\}_{i \in I}$.
- A local language L_i is a set of formulas defined by using certain "formation rules".

Formation Rules (FR)

- First-order formulas--wff's (predicates):
  1. Atomic formulas: A, B,..., P, Q,...; $\bot$, $\top$.
  2. $\neg P$, $P \wedge Q$, $P \vee Q$, $P \rightarrow Q$ for all wff's P, Q.
  3. $\forall x P(x)$, $\exists x P(x)$ for all wff's P, variables x.

  Notation for wff's: Greak letters ψ, θ, φ, ...
- Contextual formulas. For each $i \in I$,

  4. $i : \psi$ (former notation 1993-94: $< \psi, i >$).

Examples (for context i)

- Contextual Laws for $\wedge$, $\neg$ and $\rightarrow$:
  1. $i : (A \wedge \neg B) \rightarrow \neg (A \rightarrow B)$
  2. $i : \neg(A \rightarrow B) \rightarrow (A \wedge \neg B)$
- Contextual Pierce's law: $i : ((A \rightarrow B) \rightarrow A) \rightarrow A$
- Contextual De Morgan's laws:
  1. $i : \neg(A \vee B) \leftrightarrow (\neg A \wedge \neg B)$
  2. $i : \neg(A \wedge B) \leftrightarrow (\neg A \vee \neg B)$

Contextual Languages and Theories

- Σ+FR define a multicontext language $\{L_i\}_{i \in I}$.
- A set of closed wff's ($\{L_i\}_{i \in I}$-sentences) is a multicontext theory.

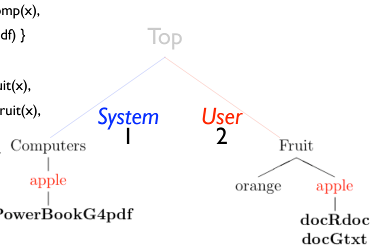
Remark: A first order theory T is a special case of a contextual theory T_i for any $i \in I$ such that a formula $\psi \in T$ iff $i : \psi \in T_i$.

- A first-order theory T_i is called local theory.

Example

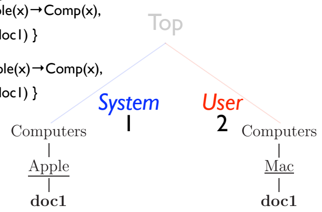
- T_{System} = {
  1: $\forall x$.apple(x) $\rightarrow$ Comp(x),
  1: apple(docPBG4pdf) }

  T_{User} = {
  2: $\forall x$.apple(x) $\rightarrow$ Fruit(x),
  2: $\forall x$.orange(x) $\rightarrow$ Fruit(x),
  2: apple(docRdoc),
  2: apple(docGtxt) }



Example

- T_{System} = {
  1:∀x.Apple(x)→Comp(x),
  1: apple(doc1) }
  T_{User} = {
  2:∀x.Mpple(x)→Comp(x),
  2: apple(doc1) }



Semantics

- A goal of semantics for modeling purposes is to model reasoning as logical consequence over a multicontext language.
- The semantics of CXL is called 'local model semantics' (LMS).
- LMS is due to Ghidini & Giunchiglia (2001)
- Ghidini and Giunchiglia give two principles for a "good" semantics... see the next slide.

Principle of Locality

- Contextual reasoning is founded on two principles - first principle is:
  - **Principle of Locality.** Reasoning uses only a part of what is potentially available.
    - The part being used while reasoning is what we call a context, i.e., a theory T_i
- The principle is already a basic intuition in Giunchiglia's work (EP 1993).

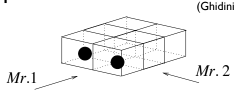
Principle of Compatibility

- Contextual reasoning is founded on two principles - second principle is:
  - **Principle of Compatibility.** There is compatibility among the kinds of reasoning performed in different contexts.
- Local Models Semantics formalizes the two principles of locality and compatibility.

Example 1 (Viewpoints)

- Observers Mr.1, Mr.2 cannot distinguish the box's depth.

(Ghidini & Giunchiglia AIJ 2001)

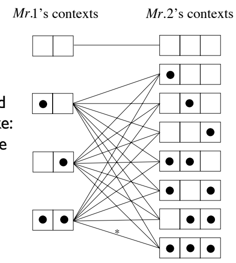


We can describe the situation by listing all possible compatible pairs:

(see next slide)

Example 1 (cont') (compatible pairs)

- Compatible pairs (see Figure) can be described by using descriptions like: "if Mr.1 sees at least one ball then Mr.2 sees at least one ball" (p. 224)

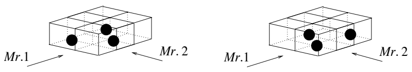


(Ghidini & Giunchiglia AIJ 2001)

Example I (cont') (Viewpoints)

- Consider the situation depicted here

(Ghidini & Giunchiglia AIJ 2001)

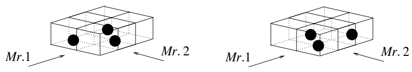


- These two different situations cannot be distinguished by the two observers.
- There's a unique compatible pair (what is it?)

Example I (cont') (Viewpoints)

- Consider the situation depicted here

(Ghidini & Giunchiglia AIJ 2001)



- To obtain a complete description of the box, we need a third view from the top.
- Mr.1 & Mr.2 can't distinguish the box's depth.

Example 2 (Beliefs)

- Let consider the beliefs an agent *a* has about world (Ghidini & Giunchiglia 2001).
- Beliefs are organized in a chain of contexts, say C(*a*), C(*aa*), C(*aaa*), etc., where:
  - C(*a*) is the root context, which represents the beliefs of *a* about a given world W.
  - C(*aaa*) formalizes the *a*'s beliefs about *a*'s beliefs about *a*'s beliefs about W, etc.

Local Models and Compatibility Sequences

- Let Mod(L_i) denote the class of first-order L_i-structures (or models for L_i).
- An element m ∈ Mod(L_i) is a local model.
- A compatibility sequence (for L= $\{L_i\}_{i \in I}$) is an

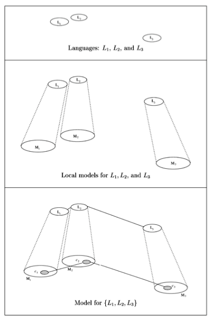
  infinite sequence **c** = $c_0, c_1, \ldots, c_i, \ldots$ where c_i is a set of L_i-structures.
- For I = {1,2} **c** is called a compatibility pair.

Models and Compatibility Relation

- Intuitively, compatibility sequences put together local models which are "mutually compatible" consistently with the situation we are modeling.
- A compatibility relation (for L) is a set C of compatibility sequences.
- A model (for L) is a nonempty compatibility relation C such that $\emptyset, \emptyset, \ldots$ is not in C.

Example of a Model

- Suppose I ={1, 2, 3}.
- We start to define languages L_1, L_2, and L_3.
- We associate each L_i with a set $M_i \subseteq$ Mod(L_i) of local models.
- We pair local models inside compatibility pairs and sequences.



(Ghidini & Giunchiglia AIJ 2001)

Remark

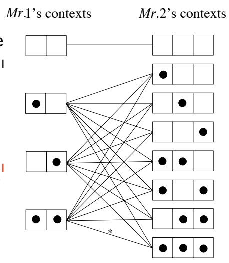
- Local models describe what is locally true.
- Compatibility sequences put together local models which are "mutually compatible" consistently with the situation we are describing (see the next example)

Example (cont') (Viewpoints - Language)

- Languages: we need languages L_1 and L_2 describing the views of Mr.1 and Mr.2.
  - What expressive power do we need?
    - For L_1 : to describe that a ball can be on the left or on the right.
    - For L_2 : to describe that a ball can be on the left, in the center, or on the right.
- Define $L_1 = \{Bl \vee Br\}$ and $L_2 = \{Bl \vee Br \vee Bc\}$.

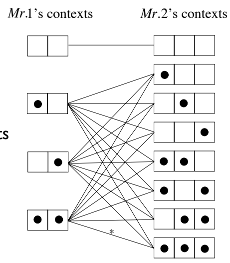
Example (cont') (Viewpoints - Models)

- Local models: we construct all the possible situations (models) for L_1 and L_2.
- This leads to the definition of the four situations (models) for L_1 (left in the figure) and of the eight situations (models) for L_2 (right in the figure)



Example (cont') (Viewpoints - Pairs)

- Compatibility pairs: we construct all the compatibility pairs.
- The figure represents all the pairs whose elements are singleton sets.
- A compatibility relation may force us to throw away some of them.



Context (Formal definition)

- Let model $C = \{c_0, c_1, \ldots, c_i, \ldots\}$ be given. A context is any c_i , namely, the set of local models $m \in Mod(L_i)$ allowed by C within any particular compatibility sequence **c**.
- Given **c**, a context captures exactly locally true facts given the constraints posed by local models of other contexts in the same compatibility sequence, as allowed by **c**.

Truth Relation (Satisfaction Relation)

- We now extend the scope of truth relation $\models_{cl}$ of FO-logic to context logic and define the truth relation $\models$ from truth relation $\models_{cl}$.
- Definition. A model C satisfies i-formula $i : \psi$ ($C \models i : \psi$), if for all $\langle c_0, c_1, \ldots, c_i, \ldots \rangle \in C$ and for all $m \in c_i$, $m \models_{cl} \psi$.
- Then C is a model of $i : \psi$, $i : \psi$ is true in C.

Truth Relation (Satisfaction Relation)

- An L_i-formula is satisfied by a model C if all the local models in each context c_i satisfy it.
- A model C satisfies a set of formulas Γ (C |= Γ) if C satisfies every formula i : ψ in Γ.
- Γ is satisfiable if C |= i : ψ for some C and for all i : ψ in Γ.
- i : ψ is valid (|= i : ψ) if C |= i : ψ for all C.

Contextual Entailment (Logical Consequence)

- A set Γ of i-formulas entails an i-formula i : ψ with respect to a model C ($\Gamma \models_C i : \psi$), if: for every compatibility sequence **c** ∈ C

  and for all j∈I with j≠i, if $c_j \models \Gamma_j$ then

  for all m ∈ c_i, if m $\models_{cl} \Gamma_i$ then m $\models_{cl}$ ψ.
- Notation: $\Gamma_k = \{\theta \mid k : \theta \in \Gamma\}$ for all k∈I.
- i : ψ is a logical consequence of Γ w.r.t. C.

Contextual Entailment (Logical Consequence)

- A set Γ of i-formulas entails an i-formula i : ψ (Γ |= i : ψ), if $\Gamma \models_C i : \psi$ for every C.
- Then i : ψ is a logical consequence of Γ.
- We can restrict logical consequence with respect to a class M of models and define $\Gamma \models_M i : \psi$ if $\Gamma \models_C i : \psi$ for all C ∈ M.
- Γ may be empty. Then i : ψ is a i-tautology.

Logical Consequence (extension of local SAT)

- **Theorem** (Ghidini & Giunchiglia 2001) Let Γ be a set of FO-formulas. For all i∈I,

  if $\Gamma_i \models_{cl} \psi$ then Γ |= i : ψ.
- This result extends classical logical consequence $\models_{cl}$ to contextual logical consequence |=.
- See (Ghidini & Giunchiglia 2001) for a proof.