# Logics for Data and Knowledge Representation

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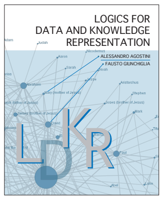
# The Logic of Defaults




- Introduction
- Language (Syntax)
- Semantics (FOL)
- Default Theories
- Reasoning Services
- Application: IHs




# Monotonic vs Non-Monotonic

- For all sets Γ of PL/FOL formulas and for all PL/FOL formulas ψ, hold:
  1. If $\psi \in \Gamma$ then $\Gamma \vdash \psi$.
  2. If $\Gamma \vdash \psi$ and $\Gamma \subseteq \Gamma'$ then $\Gamma' \vdash \psi$.
- 1. reflexivity, 2. monotonicity (Minsky, 1974) or extension property (Hayes, 1973).
- A logic where 1.+ 2. hold is called classical.




# Commonsense Reasoninig

- Intuitively, monotonicity indicates that learning a new piece of knowledge cannot reduce the set of what is known.
- Commonsense reasoning in not monotonic: Examples are:
  - reasoning by default / use of conventions
  - abductive reasoning
  - reasoning about knowledge
  - belief revision




# Two Major Types of Nonmonotonicity

- Reasoning about knowledge may be:
  - on incomplete knowledge or exceptions. This reasoning requires the use of defaults or conventions, e.g.:
    - Closed-World Assumption (CWA)
    - Negation as Failure (NasF)
  - on inconsistent knowledge. This reasoning is belief revision as special case.




# Example (Tweety)

[http://en.wikipedia.org/wiki/Tweety_Bird]

- Commonsense reasoning: most birds fly except for penguins, ostriches, etc. (*)
- Given a particular bird, we will conclude that it flies unless we happen to know that it satisfies one of the exceptions (*).
- How is the fact that *most* (but not all) birds fly to be represented?

# Example (Tweety)

- A non-monotonic deduction that Tweety does not fly is the following:
  1. bird(Tweety) [assumed]
  2. ∀x.bird(x)→flies(x) [assumed]
  3. flies(Tweety) [∀Elim+MP(1,2)]
  new knowledge → 4. penguin(Tweety) [assumed]
  5. ¬flies(Tweety)
- Consistent conclusion only withdrawing 3.





# Example (Tweety)

- The "natural" (i.e., classical, monotonic) representation explicitly lists the exceptions to flying. In first-order logic, x flies if: bird(x) ∧¬penguin(x) ∧¬ostrich(x) ∧...
- By this we cannot conclude of a 'general' bird that it can fly.
- Consider an attempt to prove fly(Tweety) assuming only bird(Tweety).





# Example (Tweety)

- Then we must establish the subgoal:

  ¬penguin(Tweety) ∧¬ostrich(Tweety) ∧...
- This is impossible given that **there is no further information about Tweety!**
- The problem is that we are blocked from concluding that Tweety can fly even if intuitively we want to deduce just that.





# Limits of Classical Logic

- The example shows some limits of classical logics (especially FOL) when applied to modeling of commonsense reasoning.
- Two major limits are known as
  1. Qualification Problem.
  2. Frame Problem.
- Both motivated attacks againts classical logics and new work in nonmonotonic logic.





# Limits of Classical Logic (Qualification Problem)

- A complete representation of knowledge makes it necessary to list all possible exceptions explicitly:

  ∀x.penguin(x) → ¬flies(x)

  ∀x.ostrich(x) → ¬flies(x)

  ... (possibly infinitely many formulas!)
- Equivalently, write the infinite formula: ∀x.(penguin(x) ∨ ostrich(x) ∨ ...)→¬flies(x).





# Limits of Classical Logic (Qualification Problem)

- This is beyond the expressivity power of FOL (infinite formulas are not allowed!)
- Even if a complete list of exceptions is available, and infinitary formulas are allowed (FOL extended), still we have to prove that Tweety is not a penguin, not a ostrich, etc.
- This is impossible, we have not a complete information on Tweety.

# Default Logic (R. Reiter, 1939-2002)

- In *A Logic of Default Reasoning* (1980): A non-monotonic logic is a formal logic whose consequence relation is not monotonic.
- In the absence of any information to the contrary, default logic assumes that reasoning patterns are a form of plausible inference, where typically conclusions must be drawn despite the absence of total knowledge about the world.




# Language (Syntax)

- From the standpoint of language, Reiter's Default Logic (DfL) has nothing new.
- The language of DfL is a (classical) first-order language., i.e. a set of formulas (wff's) over a FO-alphabet Σ.
- In addition to FO-formulas, the language of default logic employs defaults, i.e. special kinds of inference rules.




# Semantics

- In his development of default logic, Ray Reiter provided a kind of "fixed-point semantics," i.e. a characterization of the extensions of a default theory.
- A reason for this is that default logic has attracted attention mainly as a formal system, i.e., the proof-theory side of a logic for nonmonotonic reasoning.




# Semantics

- In this lecture we do not present any of the many (model-theoretic) semantics (e.g., stable models) available for default logic.
- We will focus on the original 'semantics', and present Reiter's fixed-point characterization of the extensions of a default theory as a kind of (not model-theoretic) semantics.
- We proceed mostly by examples.




# Default Reasoning with Incomplete Knowledge



# Example (Tweety, cont')

- We need to allow Tweety to fly by default.
- The default is interpreted as: 'If x is a bird, then in the absence of any other information, infer that x can fly.'
- **Problem:** How do we interpret "in the absence of any other information"?
- **Solution:** "It is consistent to assume that".

## Example (Tweety, cont')

- According to Reiter, we can rewrite

  'If x is a bird, then in the absence of any other information, infer that x can fly'

  as follows:

  'If x is a bird and it is consistent to assume that x can fly, then infer that x can fly.'




## Example (Tweety, cont')

- To model

  'If x is a bird and it is consistent to assume that x can fly, then infer that x can fly'

  we can use a 'default rule' like this:

$$\frac{\text{bird}(x) \quad : M\text{fly}(x)}{\text{fly}(x)}$$

where $M$ is read "it is consistent to assume".




## Defaults as Fuzzy Quantifiers

- In Tweety's example we defined a default to represent the "fuzzy quantifier" almost all.
- Similarly, we may define a rule to represent the dual "fuzzy quantifier few".
- For example,

$$\frac{\text{Italian}(x) \quad : M\neg\text{readbooks}(x)}{\neg\text{readbooks}(x)}$$




## General Defaults (Definition)

- A default for a first-order language L is any expression (inference rule) of the form

$$\frac{\alpha(x) \quad : M\beta_1(x), ..., M\beta_m(x)}{w(x)}\ \rho$$

where $\alpha(x)$ (prerequisite), $\beta_1(x),...,\beta_m(x)$ and $w(x)$ (consequent) are wff's of L whose variables are among those of $x = x_1, ..., x_n$, and $\rho$ is the default's name (optional).




## Closed Defaults (Definition)

- A default for a first-order language L

$$\frac{\alpha(x) \quad : M\beta_1(x), ..., M\beta_m(x)}{w(x)}\ \rho$$

is closed if all $\alpha(x)$, $\beta_1(x),...,\beta_m(x)$, $w(x)$ are closed formulas ($\alpha(x)=\alpha$, $\beta_i(x)=\beta_i$, $w(x)=w$).

(Recall that a wff is closed (L-sentence) if it contains no free variables, e.g. *course(LDKR)*.




## Normal Defaults (Definition)

- A normal default is a default of the form

$$\frac{\alpha(x) \quad : Mw(x)}{w(x)}$$

where $\alpha(x)$, $w(x)$ are FO-formulas.

- Defaults in the examples so far are normal.

# Conventions

- Conventions are special kind of defaults.
  - Example: if there is no train connection on the timetable, then there is none.
- Economical and convenient: conventions make the exchange of information efficient.
- Usually left implicit.
  - Example: no extra-note on the timetable!




# The CWA

- The Closed-World Assumption is the most known convention in database design. [Reiter, "*On closed world data bases*", 1978]
- It says that an argument ψ derives from Γ if

$$\Gamma \cup Ass(\Gamma) \vdash \psi,$$

where $Ass(\Gamma) = \{\neg\theta \mid \theta \text{ atomic}, \Gamma \nvdash \theta\}$.




# Example (Timetable)

- Let us have a train timetable (i.e. a relational database). Let T be its FO-representation.
- T is a set of formulas, precisely a deductively closed (i.e. Th(T)=T) FO-theory with relations (i.e., the timetable's rows) of the form:

  connection(x, y, t)

  (i.e.: "there is a train from x to y at time t").




# Example (Timetable, cont')

- Assume that there is no information on the timetable about trains from Trento to Milan at 10 a.m. Modulo notation, this means that

  1. connection(TN, MI, 10am) is not in T.

  That is (recall that T is deductively closed):

  2. $T \nvdash$ connection(TN, MI, 10am).




# Example (Timetable, cont')

- Question: what'd tell us a system based on T about trains from Trento to Milan at 10 a.m.?
  - Classical answer (no conventions used): ?
    - T is incomplete (as it is the timetable), no more information is available.
- Solution: make use of conventions.
  - In particular, use the CWA!




# Example (Timetable, cont')

- Assume the CWA, so define the set $Ass(T) = \{\neg\theta \mid \theta \text{ atomic}, T \nvdash \theta\}$.
- Since $T \nvdash$ connection(TN, MI, 10am), it follows that ¬connection(TN, MI, 10am) ∈ Ass(T), i.e.

  $T \cup Ass(T) \vdash \neg$connection(TN, MI, 10am)
- Using the CWA, the timetable is completed!

# Default Rule for CWA

- Definition. Let R be an n-ary predicate symbol of a FO-language L with variables $\{x_1, x_2, \ldots x_n\}$. The following default rule

$$\frac{: M \neg R(x_1, \ldots, x_n)}{\neg R(x_1, \ldots, x_n)}$$

is called closed-world default rule for R.
- The rule says one can conclude (or believe) ¬R(x, ..., x) whenever it is consistent to assume (or believe) so.





# Default Rule for CWA

- The meta-rule

$$\frac{\neg connection(x_1, x_2, x_3) \in Ass(T)}{T \cup Ass(T) \vdash \neg connection(x_1, x_2, x_3)}$$

is formalized by the (logical) default rule

$$\frac{: M \neg connection(x_1, x_2, x_3)}{\neg connection(x_1, x_2, x_3)}$$

where consistency is checked in theory $T$.





# Negation as Failure (NasF)

- Negation as Failure (NasF) is the most known convention in logic programming. [Keith Clark, "*Negation as Failure*", 1978]
- Informally, it says that the negation of an argument ψ derives from Γ if ψ does not derive from Γ (or: Γ fails to derive ψ).





# Example (Airline Flight Schedule)

- Consider a (relational) database system of an airline flight schedule, and let DB be its first-order formalization (DB = FO-theory).
- Now consider the query 'Does Alitalia flight 205 connect Milan with Rome?'
- To answer, the system will attempt to derive ("prove") a proposition representing the query, say connect(AZ205, MI, RM).





# Example (Airline Flight Schedule)

- If the proof succeeds, the system will respond "yes".
- If the proof fails, the system will typically respond "no", that is, it will derive the negation of the query

¬connect(AZ205, MI, RM).





# Negation as Failure (a meta-proof rule)

- Negation as failure can be seen as a meta-proof rule of the form

$$\frac{\nvdash \Psi}{\neg \Psi}$$

where ψ is any argument (proposition).
- This meta-proof rule was called negation as failure by Keith Clark (see Clark, 1978).

# Default Rule for Negation as Failure

- Definition. Let ψ be any classical formula (e.g. a PL- or FO-formula). The following default

$$\frac{: M\neg\psi}{\neg\psi}$$

is called negation-as-failure default rule.

- The rule says one can conclude (or believe) ¬R(x, ..., x) whenever it is consistent to assume (or believe) so.





# Default Rule for NasF (Example, cont')

- Negation-as-failure meta-proof rule

$$\frac{\mathrm{DB} \nvdash \mathsf{connect}(x, y, z)}{\neg\,\mathsf{connect}(x, y, z)}$$

is formalized by the (logical) default rule

$$\frac{: M\neg\,\mathsf{connect}(x, y, z)}{\neg\,\mathsf{connect}(x, y, z)}$$

where consistency is checked in DB .





# Default Reasoning with Inconsistent Knowledge and Beliefs



# Defaults and Belief Revision

- Defaults produce formulas that extend T.
- These formulas are interpreted as beliefs about the world formalized (not completely) by a classical, first-order theory T.
- Problem: Defaults produce many extensions of the theory T (see the next Example).





# Example (Spouse, Reiter 1980)

1. A person's hometown is that of his/her spouse:

$$\frac{\mathsf{spouse}(x, y) \wedge \mathsf{hometown}(y) = z \quad : M\,\mathsf{hometown}(x) = z}{\mathsf{hometown}(x) = z}$$

2. A person's hometown is where his/her employer is located:

$$\frac{\mathsf{employer}(x, y) \wedge \mathsf{location}(y) = z \quad : M\,\mathsf{hometown}(x) = z}{\mathsf{hometown}(x) = z}$$





# Example (Spouse, cont')

- Now suppose that Mary's spouse lives in Toronto while her employer is located in Vancouver. Then:
  - By 1, Mary's hometown is Toronto.
  - By 2, Mary's hometown is Vancouver.
- Remark 1: to believe both is inconsistent, since 'hometown' is a function!

# Example (Spouse, cont')

- Note that to believe both is inconsistent, since 'hometown' is a function!
- If first we derive Toronto then we are blocked to use default 2 and derive Vancouver, and vice versa.
- It makes sense either to believe that Mary's hometown is Toronto or that Mary's hometown is Vancouver, but not both.




# Take-away Point

- The example points out that, from the interpretation that default assumptions lead to beliefs, defaults can sanction different sets of beliefs about a not completely know or inconsistent world.
- In general, defaults can sanction different extensions of a (usually incomplete) theory.




# Defaults and Classical Theories

- Problem 1: We know defaults may produce many extensions of a classical theory.
- Problem 2: Defaults may produce no extensions of a classical theory.
- Example: Suppose the set of defaults is D = { : MP/¬P } for a proposition P. It is easy to see that D does not generate extensions.




# Defaults and Classical Theories

- Problem 3: Defaults may produce inconsistent extensions of a classical theory.
- Example: Suppose that the classical theory is T = {P ∨ Q}. Consider two defaults

  D = { : M¬P/¬P : M¬Q/¬Q}.

  D generates an extension {¬P,¬Q}. Clearly, the new theory T∪{¬P,¬Q} is inconsistent.




# Default Theories (Definitions)

- Def 1. Let L be a first-order language. A default theory of L is a pair (D,W), where:
  - D is a (finite) set of defaults on L ;
  - W is a (finite, or computable) L-theory (i.e. a set of sentences or closed wffs of L).
- Def 2. A default theory (D,W) is closed if every default in D is closed.




# Extensions of a Closed Default Theory

- An extension E of a theory W ("world"):
  1. contains W (i.e. W ⊆ E).
  2. is deductively closed (i.e. Th(E) = E).
  3. For every closed default

  $$\frac{\alpha \quad : M\beta_1, ..., M\beta_m}{w}$$

  if $\alpha \in E$ and $\neg\beta_1, ..., \neg\beta_m \notin E$ (i.e. each $\beta_1, ..., \beta_m$ is consistent with E), then $w \in E$.

# Extensions of a Closed Default Theory

- Theorem. [Reiter, 1980] Let $\Delta=(D,W)$ be a closed default theory of L and let E be a set of L-sentences.
  1. $E_0 = W$.
  2. For all $i \in N$ and all $j<m$, $E_{i+1} = Th(E_i) \cup \{w \mid (\alpha{:}M\beta_1, ..., M\beta_m/w) \in D, \alpha \in E_i, \neg\beta_j \notin E\}$.

  E is an extension for $\Delta$ iff $E = \bigcup_{i \in N} E_i$.





# Default Extensions (Examples)

- Let default theory $\Delta = (D,W)$ be defined as

  $D = \{ :MA/A, :MB/B, :MC/C \}$
  $W = \{B \rightarrow (\neg A \wedge \neg C)\}$.

  $\Delta$ has two extensions:

  $E_1 = Th(W \cup \{A, C\})$ and

  $E_2 = Th(W \cup \{B\})$.





# Default Extensions (Examples, cont')

- Let default theory $\Delta = (D,W)$ be defined as

  $D = \{ :MA/\neg B, :MB/\neg C, :MC/\neg F\}$
  $W = \emptyset$.

  $\Delta$ has one extension: $E = Th(\{\neg B, \neg F\})$.

  (To see this observe that neither A nor $\neg$A can be derived by using the available defaults in D; thus A is consistent in $E_0 = W$.)





# Default Extensions (Examples, cont')

- Let default theory $\Delta = (D,W)$ be defined as

  $D = \{ :MA/\neg B, :MB/\neg A \}$
  $W = \emptyset$.

  $\Delta$ has two extensions:

  $E_1 = Th(\{\neg A\})$ and
  $E_2 = Th(\{\neg B\})$.





# Default Extensions (Examples, cont')

- Let default theory $\Delta = (D,W)$ be defined as

  $D = \{A : M\exists xP(x)/\exists xP(x), :MA/A, :M\neg A/\neg A\}$

  $W = \emptyset$.

  $\Delta$ has two extensions:

  $E_1 = Th(\{\neg A\})$ and
  $E_2 = Th(\{A, \exists xP(x)\})$.





# Normal Default Theories

- Definition. Let L be a first-order language. A normal default theory of L is a pair (D,W), where:
  - D is a (finite) set of normal defaults on L ;
  - W is a (finite, or computable) L-theory (i.e. a set of sentences or closed wffs of L).
- Theorem. [Reiter, 1980] Every closed normal default theory has an extension.

# Application to Inheritance Hierarchies



# Motivation

- By using default logic we can give a precise formal semantics to semantic networks.
- Inheritance hierarchies (IHs) with exceptions are a kind of semantic network.
- Default logic provides a formal semantics to IHs with exceptions in the same spirit first-order logic does for inheritance hierarchies without exceptions and ontologies.





# Inheritance Hierarchies (without exceptions)

- Definition. An <u>inheritance hierarchy</u> (or network) <u>without exceptions</u> is a directed, acyclic graph composed by nodes and links;
  - nodes represent individuals and classes,
  - links represent relations with no exceptions between nodes; these links are called <u>strict</u>, or <u>monotonic</u>.
- There are several kinds of links.





# IHs without exceptions (IS-A/Not-IS-A Links)

- The most important links are IS-A links:
  'A IS-A B'
  and NOT-IS-A links:

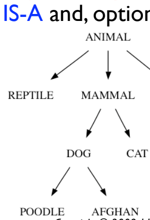
  'A NOT-IS-A B'
  for A, B be any two nodes of the hierarchy.
  - Other important links are PART-OF links.
- IHs with these links are called <u>bipolar</u>.





# Example

- An IH without exceptions is a taxonomy organized by the IS-A relation according to IS-A and, optionally, NOT-IS-A links, e.g.:




(Etherington & Reiter, 1983)





# Example (cont')

- A first-order theory $T_{IH}$ for the inheritance hierarchy IH about animals contains the following formulas (axioms) to represent subsumption of concepts and inheritance:

$\forall x\ (poodle(x) \rightarrow dog(x))$

$\forall x\ (dog(x) \rightarrow mammal(x))$

$\forall x\ (mammal(x) \rightarrow animal(x))$

# Example (cont')

- Theory $T_{IH}$ consisting of above axioms is incomplete with respect to the expected semantics of the inheritance hierarchy IH.
- Further formulas could be added to model the conventional knowledge that immediate subclasses of a node are mutually disjoint:
  ∀x (mammal(x)→¬reptile(x))
  ∀x (mammal(x)→¬insect(x)) ...




# Logical Features of IHs (1)

- Labels of nodes of an IH without exceptions are unary predicate symbols.
  - For example, ANIMAL = animal(x)
- Inheritance is a logical property
  - modeled by repeated application of *mp*:

$$\dfrac{\text{poodle(Fido)} \quad \dfrac{\forall x\ (\text{poodle}(x) \rightarrow \text{dog}(x))}{\text{poodle(Fido)} \rightarrow \text{dog(Fido)}}\ \forall E}{\dfrac{\text{dog(Fido)} \quad \dfrac{\forall x\ (\text{dog}(x) \rightarrow \text{mammal}(x))}{\text{dog(Fido)} \rightarrow \text{mammal(Fido)}}\ \forall E}{\text{mammal(Fido)}}\ mp}\ mp$$




# Logical Features of IHs (2)

- Inheritance as a *classical* (FO) logical property does not admits exceptions!
- For example, the derivation

  poodle(Fido) ⊢ animal(Fido)

  holds even if for some reason Fido enjoys other properties.




# Inheritance Hierarchies (with exceptions)

- Definition. An inheritance hierarchy with exceptions is an inheritance hierarchy whose set of links is enriched with a set of (new) links which represent relations with exceptions between nodes.
- The additional links are called default links.




# IHs with exceptions (Default-IS-A Links)

- The most important links are default-IS-A links: 'Typically A IS-A B'

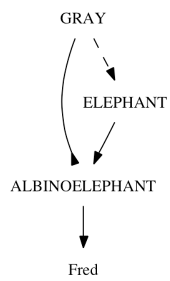
  and default-NOT-IS-A links:
  'Typically A NOT-IS-A B'

  for A, B be any two nodes of the hierarchy.
- IHs with these links are called bipolar.




# Example

- An IH with exceptions is a taxonomy organized by the IS-A relation and default IS-A relations according to IS-A/NOT-IS-A and default IS-A/NOT-IS-A links.
- The inheritance hierarchy whose semantics is given by default theory about elephants, albino elephants, and Fred, is illustrated on the right.



# Example (Tweety, cont')

- Let's consider a default theory Δ = (D,W):

$D = \left\{ \frac{\text{bird}(x) \quad : M\text{fly}(x)}{\text{fly}(x)} \right\}$,

W = {∀x penguin(x)→bird(x),

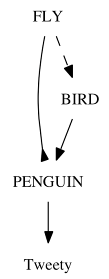
∀x penguin(x)→¬fly(x), penguin(Tweety)}.





# Example (Tweety, cont')

- The inheritance hierarchy corresponding to such default theory is illustrated on the right.








# IHs Links and Logic (Summary, 1)

We fix the correspondence between links of an inheritance hierarchy with exceptions and first-order formulas and defaults.

1. Strict IS-A: ∀x (A(x)→B(x)

(read: "A's are always B's")

2. Strict NOT-IS-A: ∀x (A(x)→¬B(x)

(read: "A's are never B's")





# IHs Links and Logic (Summary, 2)

3. Default-IS-A:

$$\frac{A(x) \quad : MB(x)}{B(x)}$$

("Typically A's are B's, with exceptions")

4. Default-NOT-IS-A:

$$\frac{A(x) \quad : M\neg B(x)}{\neg B(x)}$$

("Typically A's are not B's, with exceptions")





# Summary

- Default logic can be used to model:
  - exceptions (Ex: Tweety)
  - conventions (Ex: timetable, CWA)
  - belief revision (Ex: Spouse)
  - negation as failure (Ex: AirFlights)
  - inheritance hierarchies with exceptions





# Some Resources

- Books:

- G. Brewka, *Nonmonotonic reasoning: Logical Foundations of Commonsense*. Cambridge University Press, 1991.
- G. Antoniou and M.A. Williams, *Nonmonotonic reasoning*. The MIT Press, 1997.
- D. Makinson, *Bridges from Classical to Nonmonotonic Logic*. King's College Publications, London, UK, 2005.

- Papers & Links (if any):
  - http://dit.unitn.it/~ldkr#Biblio/