LOGICS FOR DATA AND KNOWLEDGE REPRESENTATION
Solutions of Midterm Exam of Thursday 16-04-2009

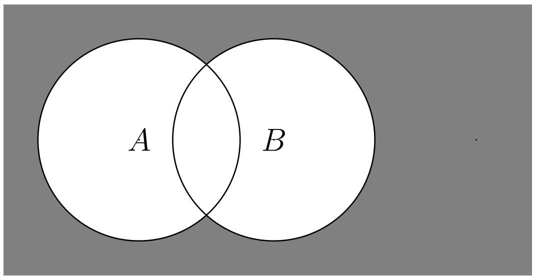
1. Write what you know about the "Levels of Formalization" in modeling of data and knowledge.
Solution: See slides. $\dashv$

2. What are the most typical reasoning tasks, or services, provided by logic? Explain and elaborate.
Solution: See slides. $\dashv$

3. What is the problem of the "semantic gap" of any representation language? Explain and elaborate.
Solution: See slides. $\dashv$

4. Describe the main steps of the DPLL procedure for deciding the SAT problem of propositional logic.
Solution: See slides. $\dashv$

5. What diagram models the extension of $(A \to B) \wedge (B \to A) \wedge \neg(A \wedge B)$?
Solution: The Venn diagram that models the extension of $(A \to B) \wedge (B \to A) \wedge \neg(A \wedge B)$ is this:



$\dashv$

6. For all formulas $p = p(x, y)$:

1. Is $\forall x \forall y\ p(x, y) \models \forall y \forall x\ p(x, y)$? yes ☐ no ☐
2. Is $\forall x \exists y\ p(x, y) \models \exists y \forall x\ p(x, y)$? yes ☐ no ☐

For each case either prove your answer or provide a counterexample.

Solution:

1. Yes. Immediate by the commutativity of 'and'.
2. No. For example, let p(x,y) be $\mathsf{Loves}(x, y)$ with the intended interpretation "person x loves person y." Then $\forall x \exists y\ p(x, y)$ means "everyone is loved by at least one person" and $\exists y \forall x\ p(x, y)$ means "there is a person that loves everyone." It is clear enough that the first sentence doesn't intuitively imply the second sentence. $\dashv$

7. 1. Represent in FOL the following database DB. In particular, (a) specify the alphabet of the FO-language L you intend to use, and (b) write the L-theory T_{DB} which models the database.

Results-LDKR

ID	Name	N.	Written	Oral	Final Mark
1.	A Jonny	128349	28		30
2.	B Gabriele	128839	20		23
3.	C Massimo	128705	27		29
4.	D Mir Shahidul	130850	27		24
5.	E Jeffrey	130882	25		30

2. Define the answer set A_q for a query q represented by the formula:

$$\exists x_1 \forall x_2 \exists x_3 \exists x_4 (\mathsf{ResultsLDKR}(x_1, x_2, x_3, 30) \vee \mathsf{ResultsLDKR}(x_1, x_2, 27, x_4)).$$

Solution: (hints) 1. $T_{DB} = \{\mathsf{ResultsLDKR}(1, A, n1, 28, -, 30), \mathsf{ResultsLDKR}(2, B, n2, 20, -, 23), ...\}$. In words, the theory is composed by all formulas of the language L that represent all rows of the table. The alphabet of L contains ResultsLDKR as 6-ary predicate symbol, no function symbols, and the following constants: $1, ..., 5; A, ..., E; n1, ..., n5; -; 20, 23, 24, 25, 27, 28, 29, 30$.

2. First observe that q is not a proper query on DB, since ResultsLDKR is a 6-ary predicate symbol, not a 4-ary predicate symbol. To proceed, we simplify the table and eliminate the 3rd and 5th column from it. By definition,

$$A_q = \{a \in Data(DB) \mid M_{DB} \models q\},$$

where $M_{DB} = (Data(DB), I)$ is a model of $T_{DB'}$ and $T_{DB'}$ is the modification of T_{DB} where every sentence is modified according to the modification of DB following the observation above. Then $A_q = \{(1, A, 28, 30), (5, E, 25, 30), (3, C, 27, 29), (4, D, 27, 24)\}$. $\dashv$

8. Translate into a suitable $\mathcal{AL}$-description logic the sentence "All students who have done at least one exam but that have not done LDKR". (Specify concepts and roles.)

Solution: We need $\mathcal{ALE}$ or $\mathcal{ALN}$. Concepts and roles are clear from the context.

In $\mathcal{ALN}$ we may write: $\mathsf{Student} \sqcap \geq 1\, \mathsf{hasdoneExam}.\top \sqcap \forall \mathsf{hasdoneExam}.\neg\, \mathsf{LDKR}$.

In $\mathcal{ALE}$ we may write: $\mathsf{Student} \sqcap \exists \mathsf{hasdoneExam}.\top \sqcap \forall \mathsf{hasdoneExam}.\neg\, \mathsf{LDKR}$. $\dashv$

9. Let AL*-concept C of the form $\leq n\mathsf{R}$ ("at-most number restriction") be given. Define the first-order formula $\tau(C)$ such that C is coherent (i.e., it has a model) iff $\tau(C)$ is satisfiable.

Solution: $\tau(C) = \forall y_1 ... \forall y_{n+1} R(x, y_1) \wedge ... \wedge R(x, y_{n+1}) \rightarrow \bigvee_{i<j} y_i = y_j$. $\dashv$

10. Are the following concepts equivalent? yes ☐ no ☐

C1. $\mathsf{Student} \sqcap \geq n\, \mathsf{hasdoneExam}$;
C2. $\leq n\, \mathsf{hasdoneExam} \sqcup \neg\, \mathsf{Student}$

Solution: No. We can translate C1 to English as "all those students who have done at least n exams." Similarly, we can translate C2 as "all those individuals or objects that have done at most n exams, or all those individuals or objects that are not students." For, C1 and C2 are clearly not equivalent. $\dashv$

11. Verify the following concept equivalences:
1. $\neg\,(C \sqcap D) \equiv \neg\, C \sqcup \neg\, D$.
2. $\neg\, \forall R.C \equiv \exists R.\neg\, C$.

Solution:

1. For all DL interpretations (Δ, I), we have the following:
$I(\neg\,(C \sqcap D)) =$
$= \Delta \setminus I(C \sqcap D)$
$= \Delta \setminus (I(C) \cap I(D))$
$= (\Delta \setminus I(C)) \cup (\Delta \setminus I(D))$
$= I(\neg\, C) \cup I(\neg\, D)$
$= I(\neg\, C \sqcup \neg\, D)$.

2. For all DL interpretations (Δ, I), we have the following:
$I(\neg \forall R.C) =$
$= \Delta \setminus I(\forall R.C)$
$= \Delta \setminus \{a \in \Delta \mid$ for all $b \in \Delta$, if $(a, b) \in I(R)$ then $b \in I(C)\}$
$= \{a \in \Delta \mid$ not for all $b \in \Delta$, if $(a, b) \in I(R)$ then $b \in I(C)\}$
$= \{a \in \Delta \mid$ for some $b \in \Delta$, not if $(a, b) \in I(R)$ then $b \in I(C)\}$
$= \{a \in \Delta \mid$ there is $b \in \Delta$ such that not either $(a, b) \notin I(R)$ or $b \in I(C)\}$
$= \{a \in \Delta \mid$ there is $b \in \Delta$ such that $(a, b) \in I(R)$ and not $b \notin I(\neg\, C)\}$.
$= \{a \in \Delta \mid$ there is $b \in \Delta$ such that $(a, b) \in I(R)$ and $b \in I(\neg\, C)\}$.
$= I(\exists R.\neg\, C)$. $\dashv$

12. A binary tree is a tree with at most two subtrees that are themselves binary trees.

1. How you represent this in DL? (I.e., write an equivalence of the form $\mathsf{BinaryTree} \equiv$ ....)
2. Define the concept "Array" in DL as a sequence of cells of length n. (Proceed similarly to 1.)

Solution: In general, there are a number of equivalent representations of the notions of binary tree and n-array (i.e., an array of lenght n). We provide one example for each notion.

1. $\mathsf{BinaryTree} \equiv \mathsf{Tree} \sqcap \leq 2\, \mathsf{hasBranch} \sqcap \forall \mathsf{hasBranch}.\mathsf{BinaryTree}$.
2. $\mathsf{nArray} \equiv \mathsf{SequenceOfCells} \sqcap \leq n\, \mathsf{hasCells} \sqcap \geq n\, \mathsf{hasCells}$.

$\dashv$