

# Reasoning about theory adequacy. A new solution to the qualification problem \*

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## Abstract

One of the main problems in commonsense reasoning is the *qualification problem*, *ie.* the fact that the number of qualifications for most general commonsense statements is virtually infinite. In this paper we argue that a solution to this problem should be based on a (meta) conjecture that the theory used to reason about the world contains all the necessary information. We also show that this *theory adequacy conjecture* can be made before the application of any of the formalisms proposed in the past, *eg.* circumscription. Finally, we present a formalization of the solution proposed using contexts and circumscription and use it to solve McCarthy's Glasgow-London-Moscow example.

## 1 The qualification problem

One of the main problems in commonsense reasoning is the *qualification problem*, *ie.* the fact that the number of qualifications for most general commonsense statements is immense and virtually infinite. McCarthy first identified this problem and explained it in the context of the Missionaries and Cannibals Puzzle (MCP) [18].

“In the missionaries and cannibals problem, a boat holding two people is stated to be available. In the statement of the problem, nothing is said about how boats are used to cross rivers, so obviously this information must come from common knowledge, and a computer program capable of solving the problem from an English description, or from a translation of

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\*A preliminary formalization of the Glasgow-London-Moscow example discussed in section 7 originally appeared in [7]. John McCarthy has provided many motivations and intuitions. Vladimir Lifschitz has given important suggestions for the axiomatization. Enrico Giunchiglia and Ben Groszof have given very useful feedback. The work described in this paper has taken advantage of the research being done inside the Mechanized Reasoning Group which aims at providing a proof theory, a semantics and a philosophical foundation to the idea of reasoning with contexts.

this description into logic, must have the requisite common knowledge. The simplest statement about the use of boats says something like, “ *If a boat is at one point on the shore of a body of water, and a set of things enter the boat, and the boat is propelled to another point of the shore, and the things exit the boat, then they will be at the second point of the shore*”. However this statement is too rigid to be true, because anyone will admit that, if the boat is a rowboat and has a leak or no oars, the action may not achieve its intended result. One might try amending the common knowledge statement about boats, but this encounters difficulties when a critic demands a qualification that the vertical exhaust stack of a diesel boat must not be struck square by a cow turd dropped by a passing hawk, or some other event that no one has previously thought of. [...]

McCarthy’s proposal is to use circumscription in an appropriate though unspecified way: “ ...if our facts don’t require that there be something that prevents crossing the river, circumscription will generate the conjecture that there isn’t ...” [20]. This solution is based on the intuition that people *jump to the conclusion* that only the mentioned qualifications are relevant. However, in [4], Ginsberg and Smith point out that the overall qualification problem consists of three distinct difficulties:

1. the language or ontology may not be adequate for expressing all possible qualifications of an action;
2. it may infeasible to write down all the qualifications (even if the ontology is adequate);
3. it may be computationally intractable to check all the qualifications.

Circumscription does not address the first difficulty and may cause problems with the second and the third. In the same paper, Ginsberg and Smith propose two new approaches which (partially) solve the second and third difficulty, but not the first. They are based on the idea that the qualifications that must be considered should be derived from the identification of some more general domain constraints which can (potentially) qualify the execution of a given action. The hypothesis underlying this and most of the formal and applied work in (nonmonotonic) commonsense reasoning is that, after all, it is sufficient to devise a unique theory including all the interesting cases. However, this does not seem satisfactory. First, it is not easy to decide *a priori* the set of facts which are relevant; indeed, it seems plausible that different qualifications are relevant in different circumstances (who would ever think of considering the event of “a cow turd in the exhaust stack” in a theory about how to use boats?). Second, people jump to conclusions not only because they explicitly discharge all the obscure and unknown qualifications, but also and simply because they do not consider all they know about the problem they are trying to solve. For instance, people know that a boat needs the oars to be propelled across a river, however this fact isn’t explicitly considered when solving the MCP.

Our intuition is that the qualification problem should be considered from a different perspective. In general, it seems impossible to build a theory which is “big” enough to consider all the interesting qualifications. No matter how hard one works at building such a theory, it will always be possible to come up with a qualification which has not

been considered. The problem is even worse than pointed out by Ginsberg and Smith, as it is not only a problem of expressiveness of the language, but also one of knowledge. Any fixed theory will never have a sufficient language nor a sufficient set of axioms to express everything which is true in the world. In the next section, we argue that a solution to the qualification problem should be constructed in two steps. The first consists of the choice of a theory which is conjectured to be adequate for the specific problem to solve, the second of using the chosen theory *as if* it contained all is known. The second step can be performed by using any of the techniques developed in the past (*eg.* those described in [20, 4]). Sections 3, 4, 5 and 6 propose a formalization of the first step using contexts and circumscription. Section 7 contains the formalization of the Glasgow-London-Moscow example. Section 8 discusses the limitations and future developments of our formalization. Finally, section 9 discusses the related work.

## 2 A solution

The world is very complex. It can be described in many different (partial) ways, at many different levels of detail and using many different languages. From this perspective, the qualification problem can be restated as the problem of *conjecturing* that a certain theory is detailed enough to solve a problem and of *revising* this conjecture once it turns out that this is not the case. The conjecturing activity can be divided in two steps:

1. first, we conjecture that a certain theory including only part of the knowledge of an agent is adequate for reasoning about a given problem;
2. second, we use the chosen theory for reasoning *as if* such theory contained all we know about the problem to solve. This can be done by the “usual” nonmonotonic reasoning.

However, one of these two conjectures can fail, causing a revision:

1. some new fact could force the reasoner to retract the conjecture that the theory is adequate for the problem (the set of considered qualifications is too small). An example is the bridge in the MCP;
2. a new fact might cause a violation of an explicitly considered qualification. This case can be handled by the usual nonmonotonic revision inside the theory. For example, in the MCP we could add the fact that there is an extra cannibal waiting on the opposite shore of the river.

The big picture of the solution we propose is as follows. We start with the formulation of a problem. Formally, a problem can be defined as a request to produce a proof of the statement  $\Gamma \vdash \alpha$ , where  $\vdash$  is the consequence relation of some formal system,  $\Gamma$  is a set of assumptions,  $\alpha$  is the goal to be proved from  $\Gamma$ .  $\Gamma$  can be empty. In planning, the proof allows us to construct a plan which should then be executed, the assumptions describe contingent knowledge, *eg.* what is true in the initial situation, the goal states what must be true after the plan has been executed. In the background, we have a commonsense knowledge base (KB) which contains the knowledge of an agent.

The KB is not a unique theory, it is structured as sets of facts, each about a particular topic. From now on, we informally call *KB contexts* these sets of facts; in section 3, the notion of context will be formalized.

Each KB context is associated to one (or more) symbol(s) which can be used to formulate problems. This constitutes an *associative memory*. We use this associative memory to state a form of *relevance claim*: if a certain symbol is associated to a KB context, then this KB context is assumed to be relevant for any problem in which this symbol appears. For instance, we can imagine that mentioning boats in the MCP would imply that the KB context(s) containing knowledge about boats will be inferred to be relevant. This exploits the implicit assumption that the statement of the problem fixes the description of the world which should be used to solve the problem. In other words, the amount of detail which should be considered. In our terminology this means that any specific problem formulation drives the choice of a theory at a given level of detail and, analogously, any specific problem reformulation allows us to confirm or revise this choice. This kind of relevance claim is quite simple. In principle, we can introduce arbitrarily complicated relevance claims. Their form may depend on many factors. In commonsense reasoning, for instance, it may depend on the confidence that people have in their ability of solving the problem, their current attitude (more or less oriented towards details), the time they have to solve the problem (taking into account more details requires more time) and so on. However, even such a simple form of relevance claim allows us to exploit the advantages of localizing the reasoning to subsets of a knowledge base (see for instance [15]).

The relevance claims and the associative memory define a space of theories. These theories are called *workspace contexts* (WS contexts from now on). A WS context is adequate for a problem if it contains *all* the assumptions of the problem and *all* the relevant KB contexts. This is what we call a *theory adequacy* requirement. Clearly, we can only conjecture that this form of adequacy actually holds. Indeed, any new fact added to a problem formulation can make it fail.

Notice that that our approach is aimed at solving the first aspect of the qualification problem described in [4]. Each WS context has a language and a set of facts which may not be adequate for expressing all possible qualifications of an action. However, the WS context adequacy is only conjectured and this conjecture can be revised. This allows us to classify the qualifications for a problem into three groups: the set of all the facts the reasoner knows about and that potentially qualify the solution; the subset of these facts that the reasoner explicitly considers in a given formulation; all the unexpected or unknown qualifications. All the previous approaches, in particular circumscription but also [4], don't distinguish between the first and the second kind of qualifications. Indeed, any qualification which is included in the theory has to be explicitly considered.

### 3 The formal framework

To formalize the solution proposed in section 2, we use the notion of *MultiContext system* (MC system) originally defined in [5]. An MC system is defined as a pair  $\langle \{c_i\}_{i \in I}, \Delta \rangle$ , where  $\{c_i\}_{i \in I}$  is a set of contexts and  $\Delta$  is a set of *bridge rules*. A context  $c_i$  is a triple  $c_i = \langle L_i, A_i, \Delta_i \rangle$  where  $L_i$  is the language of  $c_i$ ,  $A_i$  is the set of axioms of  $c_i$ ,  $\Delta_i$  is the set of inference rules, also called the deductive machinery, of  $c_i$ .  $\Delta_i$

allows us to deduce consequences in  $c_2$  by using only facts that are in  $c_1$ . Intuitively, a context can be thought of as a logical theory, presented as an axiomatic formal system, “immersed” in an MC system. From now on, in informal discussions, we use the terms “context” and “theory” as synonyms. Bridge rules are inference rules with premises and conclusions in distinct contexts. Thus, for instance, we represent a bridge rule which allows to derive a formula  $B$  in context  $C2$  just because a formula  $A$  has been derived in context  $C1$  as

$$\frac{\langle A, C1 \rangle}{\langle B, C2 \rangle}$$

where, notationally, we write  $\langle w, c \rangle$  to mean  $w$  and that  $w$  is a wff of the context  $c$ . (All these concepts, plus the notion of deduction in an MC system, are formally defined in [5].)

Let’s consider a family of MC systems, called *MCL*-systems, defined as follows:

$$MCL = \langle \{C, KB_1, \dots, KB_n, WS_1, \dots, WS_m\}, \Delta_{MCL} \rangle$$

$C$  is called the control context;  $KB_1, \dots, KB_n$  is a set of KB contexts representing different aspects of (commonsense) knowledge;  $WS_1, \dots, WS_m$  is a set of workspace contexts;  $\Delta_{MCL}$  is the set of bridge rules. The deductive machinery of all these contexts is inessential for the goals of this paper (as long as it is complete for classical first order logic); we assume that it is first order natural deduction [24]<sup>1</sup>. The language and the axioms of the KB contexts depend on the topic they are about (see examples in section 7). The language and axioms of the WS contexts are built up from the merging of KB contexts.

The control context  $C$  is a sort of meta-context which controls the flow of information among contexts. As part of its language,  $C$  contains the binary predicate *ist*, originally introduced in [21]. If  $c$  is a context different from  $C$  and  $w$  a formula, *ist*( $c, w$ ) is intuitively true in  $C$  iff the formula given as second argument is a theorem in the context given as first argument.

The intended interpretation of *ist* is captured in an *MCL* system using certain *reflection rules* [9], whose general form is the following:

$$\frac{\langle ist(c1, w), c2 \rangle}{\langle w, c1 \rangle} \quad \frac{\langle w, c1 \rangle}{\langle ist(c1, w), c2 \rangle}$$

The rule on the left is called reflection down, the other reflection up. The intuitive meaning of reflection down is that we can derive  $w$  in  $c1$  any time we have derived *ist*( $c1, w$ ) in  $c2$ . Reflection up has the obvious dual meaning. In fact they “connect” the holding of *ist*( $c1, w$ ) in  $c2$  to the holding of  $w$  in  $c1$ . We can impose different restrictions on these rules, namely on the contexts allowed as argument and on the properties of the reflected formulas. The reflections rules used in *MCL* are:

$$\frac{\langle ist(c, w), C \rangle}{\langle w, c \rangle} \quad \frac{\langle w, c \rangle}{\langle ist(c, w), C \rangle}$$

We pose the following restrictions. Both reflection up and reflection down are applicable only when the formula in the premise is a theorem of the context it belongs to, *ie.* it

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<sup>1</sup>MC systems have been implemented in GETFOL [6]. GETFOL provides commands for the definition of arbitrary MC systems. However, natural deduction is the logic defined by default in each context.

does not depend on any assumption. The reason here is mainly technical and it is due to the fact that we do not need the extra power which lifting these restrictions would give us (see [8] for a technical discussion of these issues).  $C$  is the control context and the variable  $c$  cannot be substituted with  $C$  itself. We also require that reflection down be applicable only when  $c$  is a workspace context. The motivations underlying this restriction is that we want to avoid the situation where the facts derived in the workspace (which depend on the contingent knowledge provided by the problem statement) are lifted out of the workspace and pushed into the KB contexts (that should contain only general commonsense knowledge).

The predicate *ist* allows us to define the notion of *lifting*. In our formalism, lifting a context  $c1$  into another context  $c2$  means that all the formulas which are true in the first are true also in the second (throughout this paper all free variables are intended as universally quantified):

$$liftc(c1, c2) \equiv \forall w (ist(c1, w) \supset ist(c2, w)). \quad (1)$$

This definition, together with the reflection rules, allows us to deduce in *MCL* that, if  $liftc(C1, C2)$  is a theorem of  $C$ , then any theorem  $w$  of  $C1$  is also a theorem of  $C2$ . This is the schema of the proof:

1. we instantiate the definition (1) with  $C1$  and  $C2$ ;
2. by Modus Ponens we infer  $\forall w (ist(C1, w) \supset ist(C2, w))$ ;
3. for any  $W$  which is actually a theorem of  $C1$ , we can infer  $ist(C1, W)$  in  $C$  by reflection up;
4. we prove  $ist(C2, W)$  by Modus Ponens;
5. finally, in  $C2$  we conclude  $W$  by reflection down.

## 4 Formulating WS contexts

In  $C$  we need a language for formulating problems. We use the binary predicate  $PbQual(w, p)$  to state that a formula  $w$  is an assumption of a problem  $p$ . We also write  $w = pbgoal(p)$  to say that  $w$  is the goal of  $p$ . A problem is entirely defined by its assumptions and goal:

$$p1 = p2 \equiv pbgoal(p1) = pbgoal(p2) \wedge \forall w (PbQual(w, p1) \equiv PbQual(w, p2)) \quad (2)$$

An assumption is a fact about a specific situation, or the statement that some event occurs in a specific situation. As we are using the situation calculus [23], the assumptions will have one of these three forms:  $holds(F, S)$ ,  $\neg holds(F, S)$  and  $occurs(E, S)$ , where  $F$  is a fluent,  $E$  is an event and  $S$  is some given situation. A goal is a formula of the form  $\exists s holds(F, s)$ .

We want to exploit the idea that the statement of a problem suggests which KB contexts are relevant for reasoning about it. This relevance claim is formulated by the following axiom:

$$In(sym, p) \wedge IsAbout(sym, kc) \supset RelCxt(kc, p) \quad (3)$$

$In(sym, p)$  is a decidable predicate which is true iff the symbol  $sym$  appears in the formulation of the problem  $p$ . This means that  $sym$  appears in any of the formulas used to state the assumptions and/or the goal of  $p$ .  $IsAbout(sym, kc)$  intuitively says that the KB context  $kc$  is about a topic related to  $sym$ ; this formalizes the idea of associative memory (see section 2). As requested, the relevance of a KB context for a problem depends on two factors: the appearance of a symbol in the formulation of a problem and the association between this symbol and a KB context.

For any problem  $p$ , we define a set of WS contexts whose contents depend on  $p$  itself.

$$PbQual(w, p) \supset ist(wrkc(p), w) \quad (4)$$

$$RelCxt(kc, p) \supset liftc(kc, wrkc(p)) \quad (5)$$

The function  $wrkc$  has a problem as argument and a context as value. It denotes the WS context of the problem given as argument. As we want to guarantee that two different problems get associated two different WS contexts,  $wrkc$  must be injective.

Axioms 4 and 5 provide a procedure for building a WS context for a specific problem. Clearly, if the formulation of a problem is redundant, then it can happen that useless contexts are declared to be relevant. This implies, by axiom (5), that we might actually lift more contexts than necessary. This does not seem a serious problem and it seems similar to what happens in commonsense reasoning (it is well known in psychology that putting redundant information in the statement of a problem, *eg.* of a puzzle, often sidetracks people). We may also load redundant information because a context contains facts which are irrelevant for the solution of the current problem. Again, this problem does not seem too serious, as this is always better than loading the whole knowledge base.

## 5 Theory adequacy

The question now is how we can formulate the theory adequacy conjecture. Let the formula  $RelWff(w, p)$  intuitively mean that a formula  $w$  is relevant for a problem  $p$ .  $RelWff(w, p)$  holds of the assumptions of  $p$  and of the formulas of the KB contexts which are relevant for  $p$ .

$$PbQual(w, p) \supset RelWff(w, p) \quad (6)$$

$$RelCxt(kc, p) \wedge ist(kc, w) \supset RelWff(w, p) \quad (7)$$

The theory adequacy conjecture is expressed in two steps. In the first, we make the hypothesis that there are no relevant wffs other than those which are provably relevant because of axioms (6) and (7). To achieve this we circumscribe  $RelWff$ . The result is

$$\begin{aligned}
& RelWff(w, p) \equiv \\
& PbQual(w, p) \vee \exists kc (RelCxt(kc, p) \wedge ist(kc, w))
\end{aligned} \tag{8}$$

The second step is the statement that every wff which is relevant for a problem be true in the context which is used for reasoning about it:

$$IAC(c, p) \equiv \forall w (RelWff(w, p) \supset ist(c, w)) \tag{9}$$

As we would expect,  $IAC(wrkc(p), p)$  is immediately satisfied for any problem  $p$  (as a matter of fact, axioms (4) and (5) are designed precisely to satisfy this condition). However, axiom (9) states a very general condition. As we will see, this allows us to prove the adequacy of a context for a problem  $p$  even if the context is not the WS context of  $p$ , *ie.* it is different from  $wrkc(p)$ .

Finally, the next axiom defines what it means to solve a problem inside an *MCL* system:

$$Solves(c, p) \equiv IAC(c, p) \wedge ist(c, pbgoal(p)) \tag{10}$$

## 6 Problem reformulation

A problem is reformulated by specifying that a new assumption has been added to a previously formulated problem (this approach can be easily generalized by allowing the addition of sets of new assumptions; however, the considered case is closer to the standard use of circumscription). A problem is the reformulation of another problem iff it has the same goal, there is an assumption  $q$  which belongs only to the reformulated problem, all the assumptions of the old problem still belong to the new one, and, finally, there are no new assumptions other than  $q$ .

$$\begin{aligned}
& PbRef(p2, p1, q) \equiv \\
& pbgoal(p1) = pbgoal(p2) \wedge \\
& (PbQual(q, p2) \wedge \neg PbQual(q, p1)) \wedge \\
& \forall w (PbQual(w, p1) \supset PbQual(w, p2)) \wedge \\
& \forall w ((\neg PbQual(w, p1) \supset \neg PbQual(w, p2)) \vee w = q)
\end{aligned} \tag{11}$$

Suppose we have proved  $PbRef(P2, P1, Q)$  ( $P1$  and  $P2$  are names of problems and  $Q$  is the name of a formula). Then  $PbQual(Q, P2)$  holds and we can infer  $RelWff(Q, P2)$  (by axiom (6)). The result of circumscribing  $RelWff$  now is different, because the pair  $\langle Q, P1 \rangle$  is not in the minimal extension of the predicate. In this case,  $wrkc(P1)$  is still adequate for  $P2$  provided that  $Q$  is a theorem of  $P1$ , *ie.* it is a consequence of the wffs which are relevant for  $P1$ . If this holds, we can prove  $ist(wrkc(P1), Q)$  (by reflection up) and therefore  $IAC(wrkc(P1), P2)$ . This satisfies the definition (9).

However,  $Q$  may not be derivable in the old WS context. We have two possible situations. In one case, the new assumption does not cause any new KB context to become relevant; in the other,  $Q$  contains some new symbol which is associated to a KB context  $KB_i$  which was not relevant for  $P1$ . In the first case, we cannot

prove  $ist(wrkc(P1), Q)$  and therefore we cannot conclude  $ist(wrkc(P1), P2)$ . In the second, we cannot prove  $ist(wrkc(P1), Q)$  nor  $ist(wrkc(P1), W)$  for any  $W$  of  $KB_i$ . Nonetheless, in both cases we can prove  $IAC(wrkc(P2), P2)$ .

In  $C$  we can prove some relations between the WS context of  $P1$  and that of its reformulation. For instance, the following *relevance inheritance* theorem.

$$PbRef(p2, p1, q) \wedge RelCxt(kc, p1) \supset RelCxt(kc, p2) \quad (12)$$

If a KB context is relevant for a problem, it is still relevant for its reformulation. The theorem depends on the fact that in problem reformulation we can only add new facts and therefore  $In(sym, P1)$  implies  $In(sym, P2)$  (for any symbol  $sym$ ).

From the definition (11), we know also that

$$PbRef(p2, p1, q) \wedge PbQual(w, p1) \supset PbQual(w, p2) \quad (13)$$

Moreover (12) and (13) imply that:

$$PbRef(p2, p1, q) \wedge RelWff(w, p1) \supset RelWff(w, p2) \quad (14)$$

However, this does not imply that any formula which is true in  $wrkc(p1)$  is true in  $wrkc(p2)$ . Indeed, in general problem reformulation is a nonmonotonic reasoning step and some of the consequences drawn in  $wrkc(p1)$  may not be valid in  $wrkc(p2)$ .

## 7 The GLM example. A case study

The Glasgow-London-Moscow (GLM) example, proposed by John McCarthy, is an example of “temporal projection”. It is about a trip from Glasgow to Moscow via London.

In the GLM example, there is a man who is at Glasgow and must fly from there to Moscow. He knows that there exist a flight from Glasgow to London and a flight from London to Moscow. A simple statement about air traveling says something like: “*If a flight connection between two cities there exists, an agent is in the first city airport and he has the ticket, then after flying he will be in the second city.*” But, as for MCP, this statement is too rigid. Anyone will admit that if the flight is cancelled or the agent loses the ticket at London, the action may not achieve the intended result. We might try to use our common knowledge about air traveling, but we might have some problems to include in this knowledge the fact that losing the pants would affect the successful performance of the action.

### 7.1 Using circumscription only

We start with a solution using a formalization of actions (originally proposed in [17]) using circumscription only. The particular formalization of reasoning about action used

in this section is not relevant, our approach is largely independent of it. It is needed only in order to show how the context mechanism integrates with the standard monotonic and/or nonmonotonic reasoning.

We have maximally unique names for actions, fluents, situations and truth values plus the following axioms ( $f$  stands for fluents and primitive fluents [17]; the distinction is not relevant here).

$$succeeds(a, s) \equiv \forall f (precond(f, a) \wedge holds(f, s)) \quad (15)$$

$$\begin{aligned} &noninertial(a, f, s) \equiv \\ &succeeds(a, s) \wedge (causes(a, f, true) \vee causes(a, f, false)) \end{aligned} \quad (16)$$

$$succeeds(a, s) \wedge causes(a, f, true) \supset holds(f, result(a, s)) \quad (17)$$

$$succeeds(a, s) \wedge causes(a, f, false) \supset \neg holds(f, result(a, s)) \quad (18)$$

$$\neg noninertial(a, f, s) \wedge holds(f, s) \supset holds(f, result(a, s)) \quad (19)$$

The following axioms are an extension of Lifschitz's formalism to consider events different from actions (this extension has been proposed by John McCarthy).

$$occurs(e, s) \supset outcome(s) = outcome(result(e, s)) \quad (20)$$

$$\neg occurs(e, s) \supset outcome(s) = s \quad (21)$$

$$rr(e, s) = outcome(result(e, s)) \quad (22)$$

$rr(e, s)$  is an abbreviation for the situation resulting from an action after all the events that occur after it have happened.

The following axioms describe what we know in general about flying.

$$causes(fly(x, y), at(y), true) \quad (23)$$

$$precond(at(x), fly(x, y)) \quad (24)$$

$$precond(hasticket, fly(x, y)) \quad (25)$$

$$causes(loseticket, hasticket, false) \quad (26)$$

$$causes(buyticket, hasticket, true) \quad (27)$$

$$precond(existsflight(x, y), fly(x, y)) \quad (28)$$

$$causes(cancflight(x, y), existsflight(x, y), false) \quad (29)$$

What has been described so far is general commonsense knowledge. We also suppose that the following facts hold of the initial situation.

$$holds(at(Glasgow), s0) \quad (30)$$

$$holds(hasticket, s0) \quad (31)$$

$$holds(existsflight(Glasgow, London), s0) \quad (32)$$

$$holds(existsflight(London, Moscow), s0) \quad (33)$$

Then, we can apply the following circumscription:

$$circum(Facts; occurs > causes, precond; holds) \quad (34)$$

and prove:

$$holds(at(Moscow), rr(fly(London, Moscow), rr(fly(Glasgow, London), s0))) \quad (35)$$

$$occurs(loseticket, result(fly(Glasgow, London), s0)) \quad (36)$$

then (34) does not allow us to obtain plan (35). However, in this case we can build the following new plan:

$$holds(at(Moscow), rr(fly(London, Moscow), rr(buyticket, rr(fly(Glasgow, London), s0)))) \quad (37)$$

## 7.2 Using contexts and circumscription

The solution described in section 7.1 can be “embedded” inside our formalism incrementally and without any modification of the axioms. The general commonsense facts about air traveling are structured as a set of KB contexts. In particular, we have a context *Action* which contains all the axioms for reasoning about actions (axioms (15)-(22)); the context *Flying* details what the action *fly* causes and one of its preconditions (axioms (23) and (24)); the context *Ticket* contains our knowledge about tickets (axioms (25)-(27)); the context *Flights* contains the axioms about the existence of flights (axioms (28) and (29)). All the contexts have axioms for unique names. Notice that we need to solve a problem of cross-context identification, *ie.* we must decide whether two constants belonging to two different contexts denote the same object. For the solution of the GLM example, we assume that different names in different contexts denote different objects and that the same name in different contexts denotes the same object.

The control context is *C* as described in sections 3, 4, 5 and 6. For the GLM example, we must specify only the associative memory:

$$IsAbout(at, Flying) \quad (38)$$

$$IsAbout(hasticket, Ticket) \quad (39)$$

$$IsAbout(loseticket, Ticket) \quad (40)$$

$$IsAbout(existsflight, Flights) \quad (41)$$

We do not give an associative axiom for the context *Action* because we suppose *a priori* that it is relevant in every formulation of the GLM example:

$$\forall p RelCxt(Action, p) \quad (42)$$

The goal is to prove that there exists a situation in which the agent is at Moscow. Possible assumptions are those expressed by the axioms (30), (31), (32), (33) and (36).

As a first example, let’s formulate a problem *P1* whose assumptions are that the agent is at Glasgow and that the two flights exist.

$$pbgoal(P1) = \exists s holds(at(Moscow), s) \quad (43)$$

$$PbQual(holds(at(Glasgow), s0), P1) \quad (44)$$

$$PbQual(holds(existsflight(Glasgow, London), s0), P1) \quad (45)$$

$$PbQual(holds(existsflight(London, Moscow), s0), P1) \quad (46)$$

For the sake of simplicity, in  $\mathcal{C}$  we use wffs as names of themselves and assume that the predicate  $In$  is defined on them (for instance,  $In(at, P1)$  is true). Let's suppose also that  $WS1$  is the (name of the) WS context for  $P1$  (the value of  $wrkc(P1)$ ). To prove  $Solves(WS1, P1)$ , we must first prove  $IAC(WS1, P1)$ . This is equivalent to proving that for all  $w$  such that  $RelWff(w, P1)$  we can prove  $ist(WS1, w)$ . Axioms (3), (6), (7) and (8), together with axioms (38), (41), (42), (44), (45), (46), allow us to infer the conjecture that  $RelWff(w, P1)$  iff  $w$  is one of the three assumptions of  $P1$  or a wffs of the KB contexts  $Action$ ,  $Flying$  and  $Flights$  (this is a consequence of circumscribing  $RelWff$ ).

From axiom (3) and (5) we can infer  $liftc(Flying, WS1)$  and  $liftc(Flights, WS1)$ ; from axiom (42) we can infer also  $liftc(Action, WS1)$ . By iterating the reasoning described in section 3, we can prove that all the wffs of the contexts  $Flying$ ,  $Flights$  and  $Action$  are true in  $WS1$ .  $WS1$  also contains all the assumptions of  $P1$  (by axiom (4)). This allows us to conclude that  $\forall w (RelWff(w, p) \supset ist(WS1, w))$ ; this implies  $IAC(WS1, P1)$ .

Then we can enter the context  $WS1$  and after applying the circumscription policy (34) we can infer (35) and therefore  $\exists s holds(at(Moscow), s)$ . We can reflect it up in  $C$  (obtaining  $ist(WS1, pbgoal(P1))$ ) and finally conclude  $Solves(WS1, P1)$ .

Some remarks. First,  $WS1$  is more local (less detailed) than the theory of section 7.1, as it does not consider the qualification of having the ticket (because the KB context  $Ticket$  has not been lifted). In minimizing the preconditions of the action  $fly$  we need not consider the fact that the traveler must have the ticket. However, notice that we have loaded in  $WS1$  the fact (29), which is redundant (in the sense that it is not used in the proof of the goal). Finally, in  $WS1$  we cannot reason about the event considered by McCarthy, namely  $loseticket$ . In  $WS1$  we do not have the language nor the axioms needed to talk about tickets. If we want to consider it, we need a step of reformulation.

Let us now consider a reformulation of  $P1$  where we consider the additional assumption that the agent has the ticket:

$$PbRef(P2, P1, holds(hasticket, s0)) \tag{47}$$

This reformulation implies:

$$\begin{aligned} &PbQual(holds(hasticket, s0), P2) \\ &\neg PbQual(holds(hasticket, s0), P1) \end{aligned}$$

by definition (11). It implies

$$RelCxt(Ticket, P2)$$

by axioms (3) and (39) and the fact  $In(hasticket, P2)$ . We can also prove:

$$\begin{aligned} &RelWff(holds(hasticket, s0), P2) \\ &ist(Ticket, w) \supset RelWff(w, wrkc(P2)) \end{aligned}$$

by axioms (6) and (7). As for some  $W$  such that  $RelWff(W, P2)$  we cannot prove  $ist(WS1, W)$  (eg.  $holds(hasticket, s0)$ ), we cannot prove  $IAC(WS1, P2)$ . We need therefore a reformulation of  $WS1$  (eg.  $WS2 = wrkc(P2)$ ). This reformulation contains also the new assumption and the wffs of the KB context  $Ticket$ . In  $WS2$  (which is provably adequate) we can infer  $\exists s holds(at(Moscow), s)$  and therefore  $ist(WS2, pbgoal(P2))$  (by reflection up). This implies  $Solves(WS2, P2)$ .

If now we consider a third reformulation  $P3$ , obtained from adding the assumption that the agent still has the ticket in *London*:

$$PbRef(P3, P2, holds(hasticket, result(fly(Glasgow, London), s0))) \quad (48)$$

we don't need a WS revision, because  $holds(hasticket, result(fly(Glasgow, London), s0))$  is a theorem of  $WS2$ . So we can prove

$$ist(WS2, holds(hasticket, result(fly(Glasgow, London), s0)))$$

and this allows us to conclude that all the relevant wffs, including the new assumption, are true in it. This implies  $IAC(WS2, P3)$ .

The situation is different if we reformulate  $P2$  with the assumption that the agent loses the ticket at London.

$$PbRef(P3, P2, occurs(loseticket, result(fly(Glasgow, London), s0))) \quad (49)$$

In this case, we have no new relevant KB contexts, but the new assumption is not a theorem of  $WS2$ . As for the assumption (47), we need a revision of  $WS2$ , namely a new context  $WS3$  containing the new assumption. In this case, it is clear that this revision is nonmonotonic. Indeed, the formula (35) is no longer a theorem of  $WS3$ , whereas it was a theorem of  $WS1$  and  $WS2$ . Instead we can infer (37), which allows us to conclude  $Solves(WS3, P3)$ .

## 8 Future work

We believe that the approach to the qualification problem proposed in this paper is very general and that it can be applied to the solution of other classical difficulties in formalizing commonsense reasoning, *eg.* the frame problem (the choice of a theory is also a choice of a frame of coordinate fluents). Nevertheless, the formalization given in this paper has two serious limitations. First, the nonmonotonic reasoning step inside the workspace must happen only after all the “axioms” (we call them *justified assumptions*) have been lifted in. However, this fact is not explicitly stated anywhere inside the system. A possible solution, currently under examination, is to state in the control context  $C$  an axiom which says that the appropriate circumscription axiom will hold in the workspace only when all the lifting has been done. Second, in the process of problem reformulation, we would like to infer  $\neg IAC(WS, P)$  if  $WS$  is not adequate for the problem  $P$  (in the current axiomatization, we have only that we are not able to prove  $IAC$ ). One way to achieve this is by proving that there is at least one relevant wff  $W$  for which we can prove  $\neg ist(WS, W)$ .

Our work on contexts is motivated by a *principle of locality* [5, 7]: in reasoning, we don't use all we know, but only a (sufficiently general) subset of it. The theory adequacy conjecture, for the moment, guarantees the requirement that the WS context is general enough, but does not guarantee its locality. When we prove  $IAC(WS, P)$ , we would like to express the hypothesis that  $WS$  is the smallest theory such that it is adequate for  $P$ . This again requires that we be able to minimize the contents of WS and not only the relevant wffs.

Finally, another important notion is that of subaction. Indeed, in our formalization, the approximation of WS contexts is due to the fact that some qualifications of the action *fly* are not considered in the adequate theory. However, we might need to increase the level of detail by considering some subactions of flying (*eg.* going to the airport, drinking a coffee, showing the ticket at the gate, and so on). This decomposition cannot be precompiled (it's not true that any time we go to the airport we drink a coffee); it should depend on a notion of relevance. Again, we would like to express the conjecture that the level of decomposition of an action in subactions is sufficient to reason about the problem (generality), but it's not more general than required (locality).

## 9 Related work

The ideas discussed in this paper are clearly related to those underlying the work presented in [22]. In particular, McCarthy has informally introduced and discussed the idea of lifting as an operation between contexts. In this respect, our contribution has been to provide a formalization of lifting and to use it to control the information flow between contexts.

The work discussed in this paper is based and builds on the work discussed in [10]. In particular, [10] first pointed out the need to have contexts and problems as objects of the control context and has introduced the predicate *IAC*.

The different WS contexts must be intended as *approximate* descriptions of the world. The notion of approximate theory used here is similar to that introduced in [19] (but not limited to mental qualities). However, in our approach, we exploit approximate theories to minimize the amount of facts considered when solving a problem. See [7] for a longer discussion on this issue. Our approach is a possible answer to McCarthy's request of having an AI formalism that allows the use of approximate theories "...but can go beyond them to the next level of approximation when possible and necessary ..." [18][section 5].

The idea of performing local reasoning has been proposed by Amy Lansky in various papers as a way to achieve efficiency (see for instance [13, 15, 14]). The intuitions are very similar to ours, in particular the way she uses the partitions is very similar to our idea of associative memory and relevance claim. The main difference is that in our case we reason inside the workspace while she works inside the precompiled partitions of the knowledge base. The idea of localized reasoning has also been proposed in [11]. Similarly to us he works inside a formal setting which allows for nonmonotonic reasoning. The main difference is that he is interested in preserving the nonmonotonic consequence relation. In other words he is interested in localizations where all and only the facts which can be derived globally can be also derived locally. In general this is not the case for the applications of circumscription which control theory formulation and reformulation. Finally, [2] introduces the notion of scope in nonmonotonic reasoning. In the case of circumscription, this amounts to applying minimization only within the extension of a predicate representing the scope of interest. Our formalism allows us to obtain a similar result by applying circumscription inside WS contexts. Any WS context has a narrower ontology than the whole knowledge base, and it is also possible to restrict the set of properties and facts taken into consideration. In this perspective, relevance claims and the associative memory define proof theoretically the scope of the

(monotonic and nonmonotonic) reasoning. They provide a possible answer to the request that "...it should be possible to determine scope from the current context, attention, and goals of the agent ..." [2].

The idea of linking the theory formulation to the problem formulation can be found in some work done in abstraction [1, 12]. In this work, the problem statement is used to drive the construction of abstract spaces, but not of the ground space, as it happens here. Another interesting similarity can be found with the notion of mental space introduced by Fauconnier [3]. The notion of mental space seems somehow similar to the notion of context. Moreover, in both Fauconnier's and our work the input is used to shape a workspace. However in his work the mental space is filled with information extracted from the natural language input, and not from the KB, as it happens here.

The notion of relevance has been extensively used and discussed in the literature, one example being [16]. In this work, similarly to what happens here, the relevance claims are declaratively stated in a metatheory. The main difference is that here the relevance claims control theory formulation while in [16] they control search inside a preexisting search space. We are not aware of any work which uses the notion of relevance to control theory formulation.

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